New approach for Accurate Prediction of Eddy Current Losses in Laminated Material in the Presence of Skin Effect with 2D FEA

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The integrated iron loss models in FE software overestimate the eddy current losses at high frequencies due to the presence of the skin-effect. This paper presents a simple detour for accurate prediction of eddy current losses in laminations with two-dimensional (2D) finite element analysis (FEA). An experimental set-up along with the simulation models is used to demonstrate the validity of the method. The utility of this method in loss separation and identification of loss coefficients is also illustrated.

Index Terms— **Eddy currents, finite element analysis, magnetic losses, skin effect**

I. INTRODUCTION

2D FE models of laminations over predict the eddy current losses by ignoring the skin effect at high frequencies. They normally assume the laminated core as non-conductive material, therefore the eddy currents are ignored in the field solution. Eddy current losses are estimated in the post processing stage by iron loss models integrated in the 2D FE assuming uniform distribution of flux density across the thickness of lamination. As an improvement, approximations may be used to take account of the eddy current effect in the field solution as in [1]. Accurate predictions are also possible with 3D FE [2], and coupled 2D and 1D FE [3], however, their implementation requires more effort than conventional 2D FE method. As a new approach, this paper shows that the laminations can be considered as conductive material in 2D FE, and by a simple modification of their resistivity, eddy current losses can be accurately predicted in many lamination geometries.

Magnetic loss coefficients in laminations are normally identified by measurement on Epstein frames or toroids. Therefore, simulation models of two laminated toroids along experimental data will be used to validate the new approach.

II. EDDY CURRENT LOSSES AND SKIN EFFECT

The iron losses in 2D FE models are usually estimated in the post-processing stage with the Bertotti three-term expression or a variation of it [4]:

$$P_{tot} = K_h f \hat{B}^2 + \frac{d^2}{12\rho}\int\left(\frac{dB}{dt}\right)^2 + K_e\int\left(\frac{dB}{dt}\right)^{\frac{3}{2}} \tag{1}$$

where $\hat{B}$ is the peak flux density, f the frequency, ρ is the lamination resistivity and K_e is the coefficient for excess losses. This usually results in a good approximation of the losses as witnessed by the widespread use of the Bertotti loss separation formula. However beyond a certain frequency, the Eddy current loss term in (1) is no longer valid due to skin effect. This critical frequency is related to penetration depth (skin depth) of the lamination which is defined by:

$$\delta = \sqrt{\frac{\rho}{\pi\mu f}} \tag{2}$$

where μ is the absolute permeability of the lamination. Beyond this critical frequency, and assuming uniform permeability, the eddy current loss densities in the frequency domain should now be rewritten as [1]:

$$P_{eddy} = \frac{\pi^2 d^2 \hat{B} f^2}{2\rho}\left[\frac{\delta(\sinh(\frac{d}{\delta}) - \sin(\frac{d}{\delta}))}{d(\cosh(\frac{d}{\delta}) - \cos(\frac{d}{\delta}))}\right] \tag{3}$$

III. LAMINATION CONSIDERED AS CONDUCTIVE MATERIAL

Two single lamination cores were considered for the simulation. Both have a width of w and a thickness of d as shown in fig. 1. If both models have the same average magnetic path (l_{avg}) in the 2D plane and $\frac{l_{avg}}{2\pi} \gg w$, they could be considered as equivalent with a good approximation.

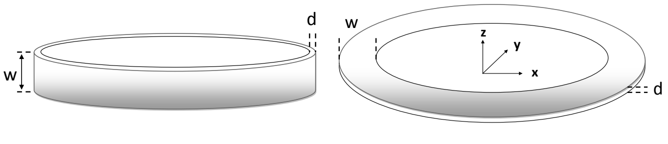

Fig. 1. Two simulated toroid cores (single lamination)

Under identical excitation, the first core can predict the losses as expected. However, the simulation results for the second core show exaggerated eddy current losses. In order to understand the differences between the two models in the 2D simulation plane, we consider the lamination strip in fig. 2 in which the eddy current path in the lamination is divided into n paths. Each path has a width of Δx=w/n in the x axis and Δz=d/n in the z axis.

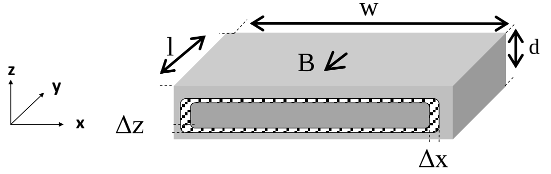

Fig. 2. Eddy current path in a single lamination

When modeling the lamination in the yz plane the resistance in the z axis is ignored. This can be considered a reasonable approximation as $w \gg d$ and eddy current losses can be accurately predicted with simulation or calculated by analytical models [5]. When the lamination is modeled in the conventional way, i.e. in the xy plane, the resistance in the x direction is ignored as the eddy current can only flows in the z direction. Since the voltage induced will be identical in both cases, the eddy current losses will be largely overestimated in the xy plane modeling. The ratio of the resistances seen by the eddy current in x and z axes is:

$$\frac{\Delta r_x}{\Delta r_z} = \frac{\rho\frac{2w}{l\Delta z}}{\rho\frac{2d}{l\Delta x}} = \left(\frac{w}{d}\right)^2 \tag{4}$$

This means that in order to have equal losses in both cases, we should choose the resistivity in the z axis as:

$$\rho_z = \rho_x \left(\frac{w}{d}\right)^2 = \rho \left(\frac{w}{d}\right)^2 \tag{5}$$

With this simple modification, we can now simulate the lamination sheet as a massive conductor. This is easy to apply in the case of an Epstein frame or a toroid. However, as the new resistivity depends on the width of the lamination, for more complex shapes like the stator of a machine, we need to divide it into 2 or 3 regions of equal width (one for back iron, one or two for the teeth).

IV. EXPERIMENTAL SETUP AND SIMULATION SCENARIOS

A laminated toroid is used to measure losses. It has 50 turns in its primary and secondary windings and its core consists of 20 laminations. The core material is M15 gauge 29 with ρ=0.52 X 10^{-6} Ω.m. The primary winding is supplied with a sinusoidal voltage source by cascading a waveform generator and a RF amplifier (10 kHz-1MHz). The losses are measured with a wide band power analyzer whose inputs are the primary input current and the secondary voltage. In this way only magnetic losses are measured. The range of the operating frequencies considered are those of the harmonics of a PWM voltage converter with 15 KHz switching frequency (15-75 kHz). The range of the measured loss densities are as high as146 W/kg. For the two simulation scenarios, the same voltage source is applied to the primary winding in the two cases (fig. 1). For each operating point in the experimental test, the permeability of the lamination is varied until the magnetization current equals that of experimental test for the same frequency and secondary voltage.

V. RESULTS AND DISCUSSION

The two simulation scenarios result in equal eddy losses in the lamination. As an additional verification, the eddy current losses in scenario 2 were also compared to those calculated by (3) at each operating point using the permeability identified with the simulation at each operating point. The results for f=30 kHz are shown in fig. 3 along with those calculated with eddy current loss term in (1). The results in fig.3 are an indication that the skin effect phenomena has been very well reproduced in the lamination and the eddy current losses are accurately estimated. It should be noted that (3) could not predict the eddy current losses if the permeability at each operating point is not accurately identified with the 2D FEA. If the eddy current loss separation and the identification of loss coefficients are accurately done, then the loss coefficients can be also applied in the post-processing stage (in the conventional way, assuming the lamination as non conductor).

Fig. 4 shows the results of losses measured by experimental test and the calculated losses with the following expression in the frequency domain:

$$P_{tot} = K_h f \hat{B}^2 + (k_1 \ln(f) + k_2)\frac{d^2 f^2 \hat{B}^{k3}}{6\rho} + (k_4 \ln(f)) + k_5)(f\hat{B})^{\frac{3}{2}} \tag{6}$$

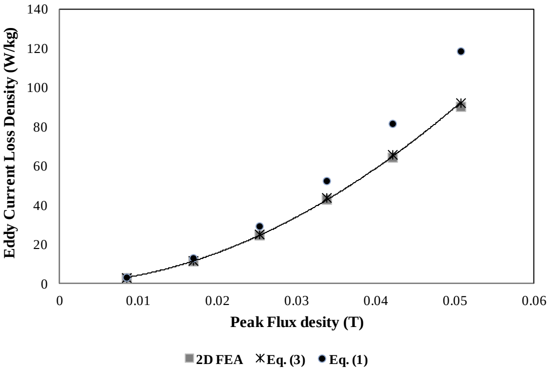

Fig.3 Eddy current losses predicted with 2D FEA ,(3), and (1) at 30 kHz

To identify the loss coefficients in (6), the hysteresis losses were estimated with their low frequency coefficient as they are not significant at these frequencies. The excess losses were found by subtracting the eddy current and hysteresis losses from the total losses measured by the experimental test. This is in contrast to the conventional methods which use total losses to identify all coefficients and therefore do not necessarily reflect the actual physical phenomena. The eddy current coefficients (k_1-k_3) were obtained by fitting with the data in fig. 3 and the excess loss coefficients (k_4-k_5) were obtained by fitting with excess loss data after separation.

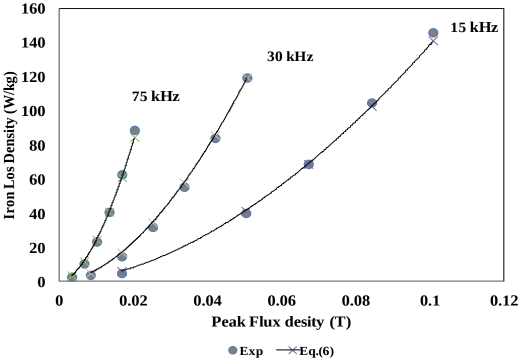

Fig.4 Measured and calculated iron losses

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